Journal of Applied Research in Water and Wastewater
E-ISSN: 2476-6283
Journal homepage: https://arww.razi.ac.ir

Experimental evaluation of discharge coefficient and performance of sloped and Non-sloped rectangular and trapezoidal piano key weirs

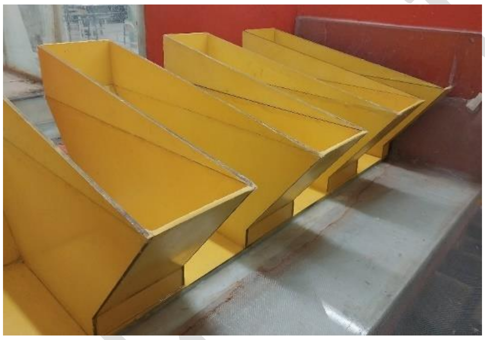
Morteza Shokri*, **Ali Gholami**

[1]*Department of Civil Engineering, Faculty of Engineering, Bu-Ali Sina University, Hamedan, Iran.*

GRAPHICAL ABSTRACT

Sloped Piano key weirs

***Flow discharge proportional to upstream head:**
- Low discharge at low flow rates (to store water and increase upstream head)
- High discharge at high flow rates (enhancing discharge capacity by reducing flow interference)



ARTICLE INFO

Article type:
Research Article

Article history:
Received xx Month xxx
Received in revised form xx Month xxx
Accepted xx Month xxx
Available online x Month xx

Keywords:
Discharge coefficient
Rectangular Piano key weir
Trapezoidal Piano key weir
Sidewall slope
Weir efficiency

Publisher: Razi University

ABSTRACT

The discharge of sloped piano key weirs (PKWs) is proportionate to the upstream head, which allows for improving hydraulic performance under various conditions. This study experimentally investigates the discharge coefficient and hydraulic performance of sloped and non-sloped PKWs with rectangular and trapezoidal sections. The goal was to evaluate the effects of the crest slope of sidewalls on the discharge coefficient and performance of a PKW. Tests were performed in an experimental flume with a length of 15 m and a height of 60 cm. Rectangular and trapezoidal type-A PKWs with 0° and 15° inclinations were studied. This study also incorporated the effect of the Weber number on the discharge coefficient of PKWs by evaluating hydraulic data. It was found that sloped rectangular and trapezoidal PKWs, where over 75% of the crest length contributed to the discharge, had larger discharge coefficients than their non-sloped counterparts. Moreover, in light of their larger effective length, sloped trapezoidal PKWs had a 13% greater discharge coefficient than sloped rectangular PKWs on average. This suggests that sloped PKWs could improve flood management and water storage efficiency.

1. Introduction

Nonlinear labyrinth weirs are a type of spillway that raises the discharge over their fixed width through an increased crest length. Piano key weirs (PKWs) have been developed as a new, specific type of labyrinth weirs (Machiels *et al.*, 2011). A number of major dams with PKWs were studied in France, and it was found that the jet flow over the PKW was affected by suction at low hydraulic loads, enhancing the discharge. An increased hydraulic load over the PKW reduced its performance. Furthermore, in the reconstruction project of the Barrage du Pas de l'Etroit dam, its hydraulic performance was found to have improved by 15% upon using reflector walls (Leite-Ribeiro *et al.*, 2009). The most optimal inlet-outlet key width was proposed to be above 1 (Machiels, 2012). Anderson and Tullis (2012) reported that the PKW discharge coefficient increased with the $\frac{W_i}{W_o}$ ratio. Side slopes had remarkably higher contributions than the key slope to PKW performance, with upstream slopes being much more influential than downstream slopes. Cicero *et al.* (2013) experimentally and numerically studied the hydraulic performance of trapezoidal PKWs. They found that trapezoidal PKWs had a nearly 5-20% greater discharge coefficient than their rectangular counterparts. Machiels *et al.* (2013) argued that enabling an optimal height was the largest contribution of reflector walls. Afazlian and Ahadiyan (2015) evaluated rectangular PKWs sloped in the flow and counter-flow directions with the inclinations of 3°, 5.5°, and 8°. They found that non-sloped PKWs had higher discharge coefficients than sloped and ogee weirs under a given hydraulic load and height. They also reported that an increase in the sidewall slope shortened the PKW length that contributed to the discharge, leading to flow interference and decreasing the discharge coefficient. It was also found that PKWs sloped in the counter-flow direction had a larger discharge than the PKWs sloped in the flow direction. Yar and Ahadiyan (2016) argued that flow interference over sidewalls is a solid fact regarding rectangular PKWs and lowers their performance. Parapet walls would reduce the flow interference over rectangular PKWs and enhance their performance. Belzner *et al.* (2017) reported that rectangular type-A PKWs displayed larger discharge coefficients than type-C PKWs. Rezaei-Ahvanooei *et al.* (2019) investigated the hydraulic performance of PKWs and found that an increase in the upstream hydraulic load induces flow interference and localized

---

*Corresponding author Email: m.Shokri@basu.ac.ir, msh.shokri@gmail.com

Cite this article: M. Shokri, A. Gholami, Experimental evaluation of discharge coefficient and performance of sloped and Non-sloped rectangular and trapezoidal piano key weirs, *Journal of Applied Research in Water and Wastewater*, xx (x), 2025, xxx. doi: https://doi.org/10.22126/arww.2026.11468.1350

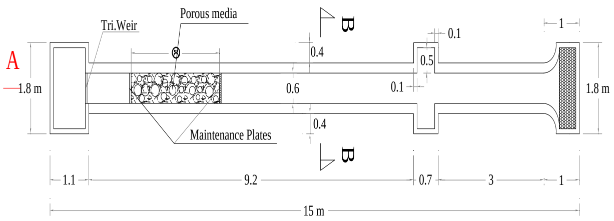
submergence, decreasing the discharge coefficient. Kumar *et al*. (2020) experimentally and numerically studied and compared the discharge coefficients of rectangular and trapezoidal PKWs and found that trapezoidal PKWS showed higher performance than their rectangular counterparts. Sangsefidi *et al*. (2021) studied the discharge coefficients of trapezoidal and rectangular PKWs, reporting the former to have a nearly 5% greater discharge coefficient than the latter. Kheir-Abadi *et al*. (2020) analyzed the effects of the reflector wall configuration on the discharge of type-D PKWs. They demonstrated that parapet walls positioned on sidewalls and inlet keys (S_2) enabled a larger discharge coefficient than configurations where they were positioned merely on the sidewalls (S_3) or on the entire crest (S_1). Sohrabzadeh and Ghodsian (2022) experimentally studied the effects of the sidewall slope on the discharge of triangular PKWs. They indicated that the upstream level in such PKWs with a sidewall inclination of 10° was higher than that in conventional triangular PKWs, while the effective hydraulic load was lower over the sloped PKW than over the non-sloped PKW. It was also found that the sloped PKWs had greater discharge coefficients than their non-sloped counterparts. Ghodsian and Sohrabzadeh Anzani (2023) compared discharge coefficients of experimental rectangular PKW models with $\frac{B}{W}=\frac{1}{3}$ and 0° and 20° crest inclinations. They performed tests at flow rates of 50-180 L/s. They demonstrated that the sloped PKW model outperformed the non-sloped PKW model.

Shaker *et al*. (2023) evaluated the effect of parapet wall geometry on the performance of rectangular PKWs and demonstrated that triangular and trapezoidal parapet walls would lead to remarkably higher discharge performance compared to rectangular parapet walls. Sohrabzadeh and Ghodsian (2024) positioned a triangular parapet wall with a 10° slope on a portion of the crest length in rectangular PKWs and evaluated the discharge coefficients of the modified and unmodified PKWs. They showed that the PKW with a parapet wall had a higher discharge coefficient than the PKW with no parapet wall. They also found that the PKW with a parapet wall had higher efficiency than its counterpart with no parapet wall. Alabedi and Khassaf (2024) studied the hydraulic performance of type-B PKW and found that a PKW model with an $\frac{L}{W}$ ratio of 6 had the highest performance, whereas a decrease in the $\frac{L}{W}$ ratio to 3 and 4 dramatically decreased performance.

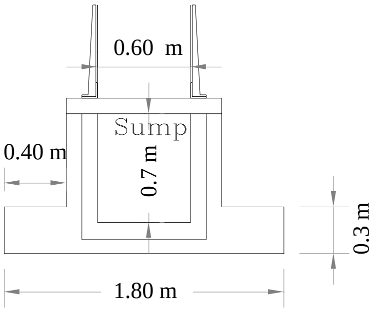
The literature on sloped and non-sloped rectangular PKWs provides contradictory results. For example, Afzalian and Ahadiyan (2015) examined sloped rectangular PKWs and found that non-sloped PKWs had higher discharge coefficients. On the other hand, Ghodsian and Sohrabzadeh Anzani (2023) compared sloped and non-sloped rectangular PKWs, reporting that sloped PKWs would display larger discharge coefficients. This contradiction and the flow rate range studied by Ghodsian and Sohrabzadeh Anzani (2023) suggest that the behavior of sloped rectangular PKWs remains to be comprehensively evaluated. This study compares rectangular and trapezoidal PKWs with 0° and 5° slopes over a flow rate range of 3.25-72.62 L/s to capture their performance similarities and differences under a given set of conditions and provide practical implications to contribute to more effective use of such PKWs in hydraulic projects. Dams and irrigation and drainage networks are among the applications of sloped PKWs. PKWs play a key role in dams since they allow for higher water storage without raising the dam height during non-critical periods, enabling a more cost-effective system. PKWs are also used in irrigation and drainage networks to ensure the required water level during low-flow seasons.

2. Materials and methods

The tests were performed in the hydraulic laboratory of the Faculty of Engineering at Bu-Ali Sina University. A flume with glass walls and bottom, a length of 15 m, and a height of 60 cm was employed. The flow rate was set to 3.25-72.62 L/s using a control valve, and water was flown from a tank below the canal into the flume. It was redirected into the tank after flowing over the PKW (Fig. 1).



(a)



(b)

Fig. 1. The Experimental setup: (a) Schematic representation of the experimental flume plan, (b) B-B cross-section view.

Rectangular and trapezoidal type-A PKWs with a thickness of 5 mm and four cycles were designed by developing models and setting a 5° slope in AutoCAD. This inclination was set based on experimental reports to improve discharge performance. Then, components were fabricated using 5mm-thick PVC sheets through laser cutting and were assembled using an instant adhesive (123 Glue, Iran). The PKW models were placed and sealed using a silicon adhesive in the final one-meter section of the flume, where the width was widened to 97 cm using an iron plate divergent transition. The flow depth was measured using a point gauge with a precision of ±0.1 mm at a distance twice the PKW height due to the small water curvature. The PKW had been sloped, and a steady-state flow was established (with minimal flow fluctuations), marking the sidewalls as references. The marks represented the maximum flow over the sidewalls at a given flow rate. Once these marks had been implemented and stabilized, the flow was interrupted to calculate the wetted length over the sidewalls without flow interference using a ruler with a precision of ±0.1 mm. Before a sloped PKW is fully activated, the sum of the wetted length of the sidewalls and the wetted length of the inlet or outlet keys represents the effective wetted length of the PKW, which would be used as an important parameter in the next analyses. The inclination of PKWs increases the downstream height (P_o). Table 1 describes the PKWs, namely the non-sloped rectangular PKW (RPKW0), non-sloped trapezoidal PKW (TPKW0), sloped rectangular PKW (RPKW5), and sloped trapezoidal PKW (TPKW5).

2.1. Dimensional analysis

Fig. 2 depicts the geometric parameters that affect discharge performance in rectangular and trapezoidal PKWs with sloped sidewalls.

These geometric parameters are given by:

$$F(W_i, W_o, W, B_i, B_o, B, T_s, L, g, P_i, P_o, S_o, N, H, Q, \mu, \rho, \sigma, L', \theta, \alpha, \text{Re}, \text{We}, \text{Fr})=0 \quad (1)$$

where W_i is the inlet key width (m), W_o is the outlet key width, W is the total key width, B_i is the downstream slope length, B_o is the upstream slope length, $B = B_i + B_o$ is the sidewall crest length, T_s is the PKW thickness, L is the total crest length, g denotes the gravitational acceleration, P_i is the upstream crest height, P_o is the downstream crest height, S denotes the canal bottom slope, N is the number of PKW keys, H is the total upstream head (which is $h+\frac{v^2}{2g}$), Q is the PKW discharge, μ is the dynamic viscosity, ρ is the density if water, σ is the surface tension of water, L' is the effective crest length (wetted length), θ is the sidewall inclination from the vertical axis (vertical slope), α is the sidewall inclination from the horizontal axis (horizontal slope), Re is the Reynolds number, Fr is the Froude number, We is the Weber number, and F denotes "function." Here, $\theta=tan^{-1}\frac{D}{B}$, where D is the height of the walls added to the PKW. Dimensional analysis gives:

$$F\left(\frac{Q}{L\sqrt{g}H^{\frac{3}{2}}}, \frac{W_i}{W_o}, \frac{W_i}{W}, \frac{W}{B_o}, \frac{L}{W}, \frac{B_i}{B_o}, \frac{B}{W}, \frac{H}{P_i}, \frac{W}{B_i}, \frac{T_s}{B}, \frac{T_s}{P_o}, S, N, \frac{L'}{L}, \theta, \alpha, \text{Re}, \text{We}, \text{Fr}\right)=0 \quad (2)$$

Table 1. Experimental sloped and non-sloped PKW models.

Model	S_i=S_o	θ	L (cm)	B_i=B_o (cm)	B_B (cm)	P_o (cm)	P_i (cm)	W_o (cm)	W_i (cm)	W (cm)
RPKW	53°	0°	493	12.5	25	20	20	12.5	12.5	97
		5°	494.5	12.5	25	24.4	20	12.5	12.5	97
TPKW	53°	0°	453.4	12.5	25	20	20	7.5	17.5	97
		5°	454.9	12.5	25	24.4	20	7.5	17.5	97

The prototypes would not affect surface tension in nature; however, they affect surface tension in experimental settings at low heads. A review of hydraulic data from previous experimental reports suggests that the Weber number was in two ranges (<50 and >50). Surface tension affects the discharge coefficient (Machiels, 2012). Here, Re ($\frac{\rho VL}{\mu}$) could be neglected since it was in the turbulence range. Fr was constantly below 1 upstream, and the flow was sub-critical in this study. Moreover, the $\frac{W_i}{W_o}$, $\frac{B_i}{B_o}$, $\frac{T_s}{P_o}$, $\frac{W}{B_i}$, $\frac{B}{W}$, $\frac{T_s}{B}$, $\frac{W_i}{W}$, $\frac{L}{W}$, and $\frac{W}{B_o}$ ratios were kept unchanged in the tests. It should be noted that α=2, N=4, and S=0. Excluding the fixed terms, the discharge coefficient of sloped rectangular and trapezoidal PKWs can be written as:

$$C_d = \frac{Q}{L\sqrt{g}H^{\frac{3}{2}}} = \mathrm{F}\left(\frac{H}{P_i}, \frac{L'}{L}, \theta, \mathrm{We}, \mathrm{Fr}\right) = 0 \tag{3}$$

The discharge coefficient of sloped rectangular and trapezoidal PKWs is dependent on the dimensionless parameters of We, Fr, θ, $\frac{L'}{L}$, and $\frac{H}{P_i}$. It is worth mentioning that θ and $\frac{L'}{L}$ in non-sloped rectangular and trapezoidal PKWs are 0 and 1, respectively, and are excluded from Eq. (3).

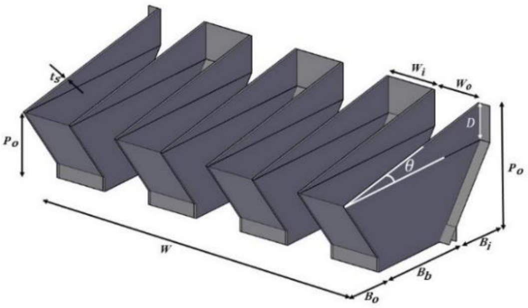


Fig. 2. Geometric parameters affecting discharge performance in sloped rectangular and trapezoidal PKWs.

3. Results and discussion

3.1. Experimental observations

The discharge of the sloped PKWs was observed to be proportional to the upstream head, as shown in Figs. 3 and 4. Since only half of the keys were involved at low discharge rates, the upstream head increased, raising the quantity of water stored behind the PKW. A rise in the discharge rate increased the effective length of both rectangular and trapezoidal PKWs and, therefore, the discharge capacity.

Fig. 3. Sloped rectangular PKW.

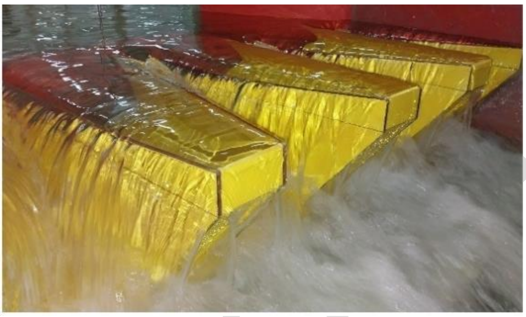


Fig. 4. Sloped trapezoidal PKW.

The sloped rectangular and trapezoidal PKWs were observed to experimentally experience flow interference in a bulging form before the activation of the inlet keys, and the height increased with the discharge rate. Fig. 5 shows the bulge over the sloped rectangular PKW.

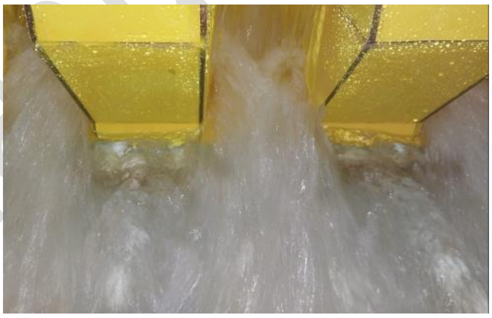


Fig. 5. Flow interference beneath the inlet keys in the sloped rectangular PKW.

Figs. 6 and 7 depict flow interference over the sloped and non-sloped rectangular PKWs at a given discharge rate. The incorporation of an inclination into a rectangular PKW would decrease flow interference and, therefore, improve discharge performance.

3.2. Effective length

As mentioned, merely half of the keys are involved in the discharge at low flow rates. Thus, the wetted-to-total length ratio ($\frac{L'}{L}$) would be a determinant of the discharge coefficient. For a sloped PKW model, a greater head is required to activate the entire crest length. The sloped trapezoidal PKW was observed to have a 4% larger effective length than its sloped rectangular counterpart on average (Fig. 8). Furthermore, the discharge coefficients of the sloped rectangular and trapezoidal PKWs were found to be greater than those of the non-sloped rectangular and trapezoidal PKWs at $\frac{L'}{L}$ ratios above 0.75.

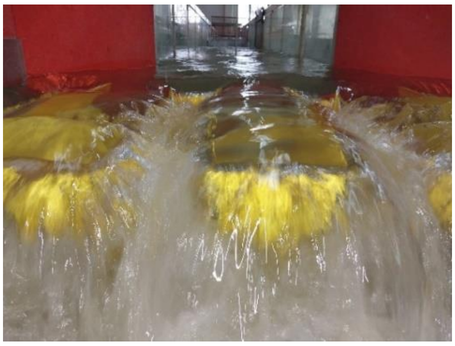


Fig. 6. Flow interference in the outlet keys of the non-sloped rectangular PKW.

Fig. 7. Flow interference in the outlet keys of the sloped rectangular PKW.

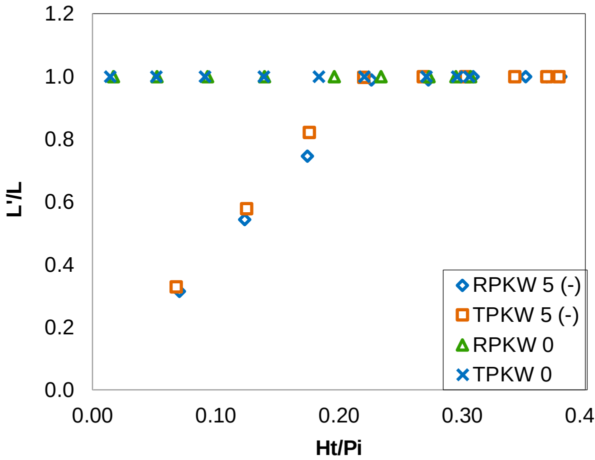


Fig. 8. $\frac{L'}{L}$ ratio versus $\frac{H}{P_i}$ ratio in the sloped and non-sloped rectangular and trapezoidal PKWs.

3.3. Discharge coefficient

The discharge coefficient of a PKW is calculated using the general weir equation:

$$C_d=\frac{Q_E}{Q_t}=\frac{Q_E}{\frac{2}{3}L\sqrt{2g}H_t^{\frac{3}{2}}} \tag{4}$$

where, C_d is the discharge coefficient, Q_t is the theoretical discharge, Q_E is the experimental discharge, L is the effective crest length, g is the gravitational acceleration, and H_t denotes the total water height over the PKW.

The effective crest length of a PKW is not a constant parameter since it increases with the head. Thus, the general weir equation is not efficient in calculating the discharge coefficient of PKWs and requires modifications.

To improve the accuracy of the formulation, the theoretical discharge is divided into five terms (Afzalian, 2014):

$$Q_t=Q_{Wi}+Q_{Wo}+Q_{SW} \tag{5}$$

where Q_t is the theoretical discharge, Q_{Wi} is the inlet key discharge, Q_{WO} is the outlet key discharge, and Q_{SW} is the sidewall discharge.

The modified formulation calculates the discharge coefficient more accurately as the PKW sidewall slope θ is incorporated. In fact, the modified formulation changes into the general weir equation when the sidewall slope is 0 (non-sloped PKW). This modified formulation has been reported to show satisfactory accuracy in the discharge coefficient calculation and discharge performance evaluation of PKWs in earlier works.

The theoretical discharge $\boldsymbol{Q_t}$ is calculated using Eq. 6 when the H/D ratio is below 1 and using Eq. (7) when the H/D ratio is above 1:

$$Q_t=\frac{2}{3}\sqrt{2g}L'_{wo}H^{1.5}+\frac{4}{15D}\sqrt{2g}L'_{sw}H^{2.5} \tag{6}$$

$$Q_t=\frac{2}{3}\sqrt{2g}L'_{wo}H^{1.5}+\frac{4}{15D}\sqrt{2g}L'_{sw}\left[H^{2.5}-(H-D)^{2.5}\right]+\frac{2}{3}\sqrt{2g}L'_{wi}(H-D)^{1.5} \tag{7}$$

Sloped PKWs are expected to show small discharge coefficients at low heads, with a rise in the head involving all the keys and enabling a larger discharge coefficient compared to non-sloped PKWs. It was found that the non-sloped rectangular and trapezoidal PKWs showed a larger discharge coefficient than their sloped counterparts at H_t/P ratios below 0.12, while a rise in the H_t/P ratio above 0.12 involved more than 75% of the crest length in the discharge and led to higher discharge coefficients in the sloped rectangular and trapezoidal PKWs than in the non-sloped PKWs. The sloped rectangular PKW was observed to have a 6% larger discharge coefficient than the non-sloped rectangular PKW, whereas the sloped trapezoidal PKW showed a 7% larger discharge coefficient than the non-sloped trapezoidal PKW on average. Figs. 9 and 10 compare the discharge coefficients of the sloped and non-sloped rectangular and trapezoidal PKWs, respectively.

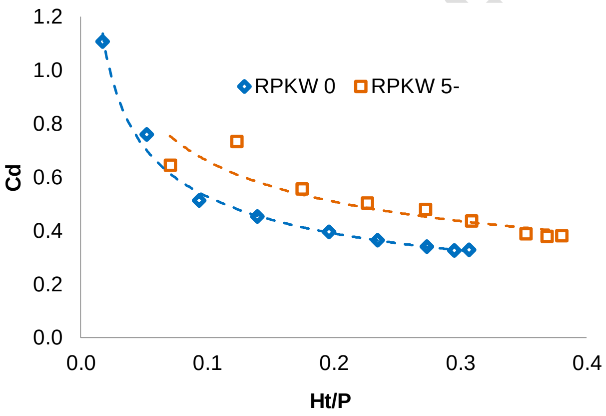


Fig. 9. Discharge coefficient of sloped versus non-sloped rectangular PKWs.

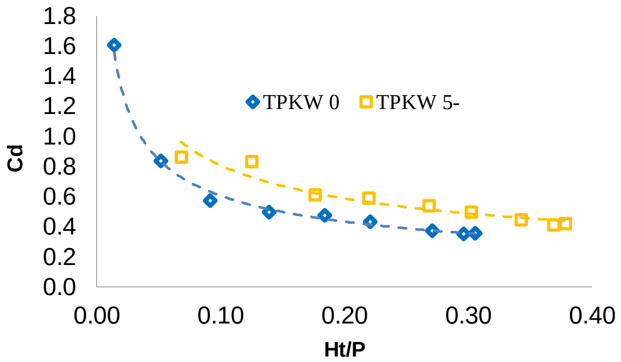


Fig. 10. Discharge coefficient of sloped versus non-sloped trapezoidal PKWs.

3.4. Discharge efficiency

The discharge efficiency of a PKW is calculated as (Sangsefidi *et al.*, 2021):

$$\varepsilon=C_d\times\frac{L}{W} \tag{8}$$

where, ε is the discharge efficiency, L is the total crest length, W is the total crest width, and C_d is the discharge coefficient. The efficiency of the sloped rectangular and trapezoidal PKWs decreased at a smaller rate than that of the non-sloped PKWs as the head increased. In other words, the sloped PKWs displayed higher performance than the non-sloped PKWs. Based on Eq. (8), the discharge coefficient is multiplied by the L/W ratio, and the sloped rectangular and trapezoidal PKWs had, on average, 6% and 7% higher efficiency than their non-sloped counterparts, respectively, as shown in Figs. 11 and 12.

3.5. Sloped rectangular PKW versus sloped trapezoidal PKW

Figs. 13 and 14 compare the sloped rectangular and trapezoidal PKWs in the discharge coefficient and discharge efficiency, respectively. The sloped trapezoidal PKT was observed to have a 13% greater discharge coefficient and 5% higher discharge efficiency than the sloped rectangular PKW on average.

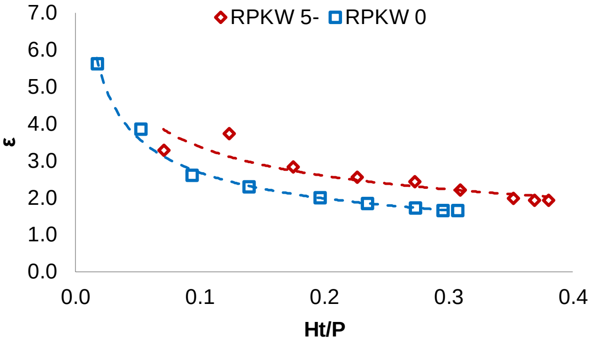


Fig. 11. Discharge efficiency of sloped versus non-sloped rectangular PKWs.

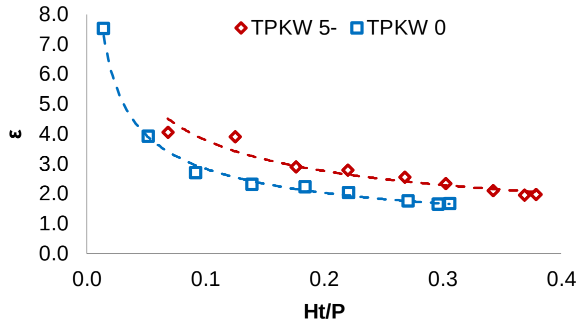


Fig. 12. Discharge efficiency of sloped versus non-sloped trapezoidal PKWs.

3.6. Discharge coefficient formulation

The discharge coefficient was argued to be a function of the dimensionless parameters of We, Fr, $tan^{-1}\theta=\frac{D}{B}, \frac{L'}{L}$, and $\frac{H_t}{P}$. Therefore, the discharge coefficient was estimated in SPSS by using two ranges to reflect the effect of the Weber number on the discharge coefficients of sloped and non-sloped rectangular and trapezoidal PWKs. Coefficients (9-1-1) and (9-2-1) are provided in Table 2 for We<50, while Coefficients (9-1-2) and (9-2-2) are provided for We>50.

$$C_d=a+\left(b\times\frac{H_t}{P}\right)+\left(c\times\left(\frac{H_t}{P}\right)^2\right)+\left(d\times\log\left(\frac{H_t}{P}\right)\right)+\left(e\times\frac{L'}{L}\right)+\left(f\times\left(\frac{L'}{L}\right)^2\right)+\left(g\times\frac{D}{B}\right)+\left(h\times\left(\frac{D}{B}\times\frac{L'}{L}\right)\right) \quad (9)$$

where, $\frac{H_t}{P}$ is the ratio of the hydraulic head over the crest to the PKW height, $\frac{D}{B}$ is the ratio of the wall height to the crest width, $\frac{L'}{L}$ is the wetted-to-total length ratio, We is the Weber number, and Fr is the Froude number.

The most optimal formulation was determined using the coefficient of determination (R^2), mean absolute percentage error (MAPE), and root-mean-squared error (RMSE):

$$MAPE=\frac{\sum_{i=1}^{n}\left|\frac{C_d\,(obs)-C_d\,(cal)}{C_d\,(obs)}\right|}{n}\times 100 \quad (10)$$

$$RMSE=\sqrt{\frac{\sum_{t=1}^{n}\left(C_d\,(obs)-C_d\,(cal)\right)^2}{n}} \quad (11)$$

where, $C_d\,(obs)$ is the observational discharge coefficient, $C_d\,(cal)$ denotes the calculated discharge coefficient, and n is the number of data samples.

Table 2. Discharge coefficients estimated based on Eq. 9.

Model	coefficient	a	b	c	d	e	f	g	h	i	j
RPKW	(9-1-1)	-1.06	-0.474	-0.886	-1.377	0.371	-0.742	-0.866	6.347	-0.008	6.434
	(9-1-2)	2.066	-4.327	2.016	1.163	-0.37	0.095	-0.194	2.171	0.899	0
TPKW	(9-2-1)	-0.464	0.45	-5.848	-1.307	-0.091	-0.261	-0.601	5.59	-0.001	0.011
	(9-2-2)	-0.364	1.872	-3.102	-1.218	-0.725	0.17	-1.673	4.188	1.211	0

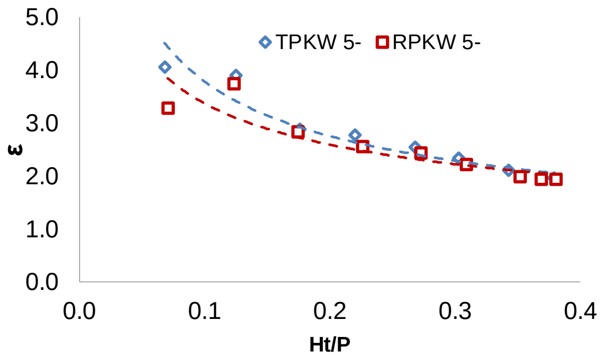


Fig. 14. Discharge efficiency of sloped rectangular versus trapezoidal PKWs.

Figs. 15 and 16 compare the proposed discharge coefficient formulation to earlier works for the rectangular and trapezoidal PKWs, respectively. The data scatter was within ±12%, suggesting that the proposed formulation has high accuracy and can calculate the discharge coefficients of both non-sloped and sloped PKWs.

Table 6 reports the sensitivity of the discharge coefficient to various parameters in PKWs (Eq. 9). The discharge coefficient was found to be more sensitive to a and $d\times\log(\frac{H_t}{P})$.

3.7. Comparison of the obtained results with literature

Machiels (2012) reported the optimal inlet-outlet key width ratio to be larger than 1. Yar and Adadiyan (2016) argued that flow interference over sidewalls is a solid fact regarding PKWs and decreases discharge performance.

The purpose of PKWs is to be considered. Non-sloped PKWs are intended to increase the discharge ratio at low flow rates. Hence, an inlet-outlet key width ratio greater than 1 could be effective. The discharge rate may decline due to flow interference as the flow rate rises. An increase in the PKW height can reduce flow interference in the outlet keys at high flow rates, increasing the discharge coefficient at both low and high flow rates. Sloped PKWs, on the other hand, are employed to decrease the discharge and store water at low flow rates by deactivating half of the keys. In PKWs sloped in the counter-flow direction, flow layer interference begins in the upstream and extends to the downstream as the hydraulic head increases.

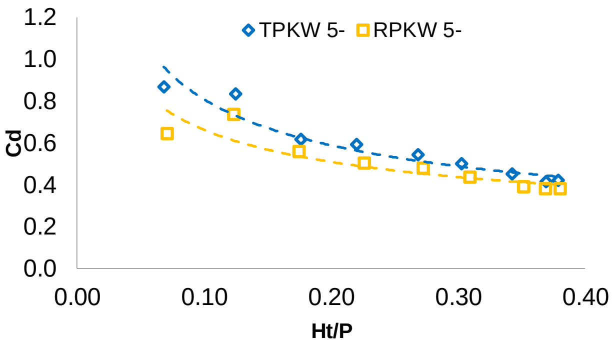


Fig. 13. Discharge coefficient of sloped rectangular versus trapezoidal PKWs.

Table 3. Coefficient of determination, MAPE, and RMSE of the discharge coefficient formulation.

Model	Coefficient	R^2	MAPE	RMSE
RPKW	(9-1-1)	0.927	7.179	0.044
	(9-1-2)	0.95	2.514	0.011
TPKW	(9-2-1)	0.947	4.792	0.065
	(9-2-2)	0.937	2.984	0.014

Therefore, flow interference in sloped PKW is an important factor since a lower outlet key width may increase flow interference and reduce the discharge coefficient. At inlet-outlet key width ratios above 1, the discharge coefficient reduces at both low and high flow rates. Hence, since the inlet-outlet key width ratio was 1.5 in Yar and Adadiyan (2016), the sloped PKWs had smaller discharge coefficients than the non-sloped PKWs. This study increased the outlet key width from 7.5 to 17.5 cm in the sloped trapezoidal PKW to avoid full flow interference in the outlet key. Moreover, the inlet-outlet width ratio was 1 in the sloped rectangular PKW. According to Figs. 6 and 7, introducing a slope to the PKW reduced flow interference and moderated the rate at which the discharge coefficient decreased. A comparison of the sloped and non-sloped spillway systems indicated that the sloped rectangular and trapezoidal PKWs, on average, had 6% and 7% larger discharge coefficients than the non-sloped rectangular and trapezoidal PKWs, respectively.

Table 6. Discharge coefficient sensitivity in PKWs.

Model	Coefficients	a	$b\times\frac{H_t}{P}$	$c\times\left(\frac{H_t}{P}\right)^2$	$d\times\log\left(\frac{H_t}{P}\right)$	$e\times\frac{L'}{L}$	$f\times\left(\frac{L'}{L}\right)^2$	$g\times\frac{D}{B}$	$h\times\left(\frac{D}{B}\times\frac{L'}{L}\right)$	i×Fr	j×We
RPKW	(9-1-1)	32%	1%	0%	48%	4%	4%	3%	6%	0%	3%
	(9-1-2)	41%	28%	4%	12%	6%	1%	0%	4%	3%	-
TPKW	(9-2-1)	19%	1%	2%	63%	1%	2%	3%	8%	0%	0%
	(9-2-2)	10%	17%	9%	18%	19%	4%	5%	11%	7%	-

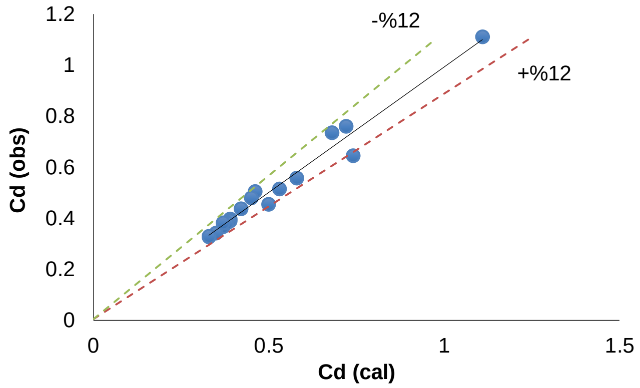


Fig. 15. Data scatter for rectangular PKWs.

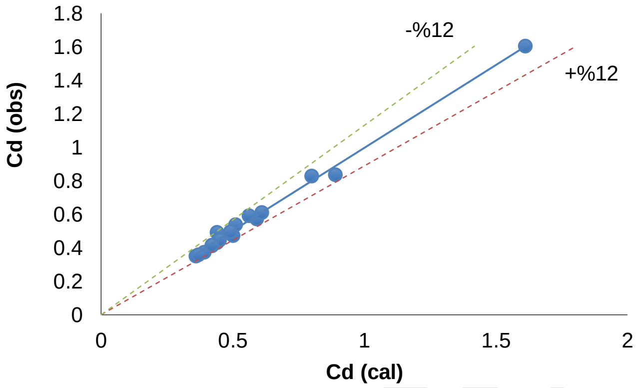


Fig. 16. Data scatter for trapezoidal PKWs.

4. Conclusions

This study experimentally evaluated the discharge coefficients and performance of sloped and non-sloped rectangular and trapezoidal PKWs. The results can be summarized as:

1. In the sloped rectangular and trapezoidal PKWs, an increase in the Ht/P ratio above 0.12 and, thus, involving more than 75% of the crest length in the discharge raised the discharge coefficient compared to the non-sloped rectangular and trapezoidal PKWs.
2. The discharge coefficient and performance of the sloped rectangular PKW were 6% higher than those of the non-sloped rectangular PKW on average, whereas the discharge coefficient and performance of the sloped trapezoidal PKW were 7% larger than those of the non-sloped trapezoidal PKW.
3. The sloped trapezoidal PKW had a 13% higher discharge coefficient and 5% higher discharge performance than the sloped rectangular PKW on average.
4. A formulation was proposed to estimate the discharge coefficients of sloped and non-sloped rectangular and trapezoidal PKWs.

Sloped PKWs are an innovative solution for water resource management and flood control and display optimal performance under various conditions in light of their unique design. However, further research is to be conducted to cope with the flow interference beneath the inlet keys in the downstream of sloped PKWs. Therefore, future studies may address downstream scouring in sloped PKWs.

Author Contributions

Morteza Shokri: Contributed to the study conception and design, conducted material preparation, data collection, and analysis, and provided feedback on prior versions of the manuscript.
Ali Gholami: Contributed to the study conception and design, conducted material preparation, data collection, and analysis, and composed the first draft of the manuscript.

Conflict of Interest

The authors have no competing interests to declare, including non-financial ones.

Acknowledgement

The authors are grateful for the support provided by the hydraulic laboratory at Bu-Ali Sina University.

Data Availability Statement

Datasets employed and/or evaluated in this research are available from the corresponding author upon justifiable request.

References

Afzalian, A.R. and Ahadiyan, J. (2015) ‘Piano key weir with sloped parapet wall’, *Irrigation Sciences and Engineering*, 38(2), pp. 91–102. doi: https://doi.org/10.22055/jise.2015.11351

Alabedi, H.J. and Khassaf, S.I. (2024) ‘Numerical study on discharge capacity of piano key side weir with various ratios of the crest length to the width’, *Open Engineering*, 14(1), p. 20220536. doi: https://doi.org/10.1515/eng-2022-0536

Anderson, R. and Tullis, B. (2012) ‘Comparison of piano key and rectangular labyrinth weir hydraulics’, *Journal of Hydraulic Engineering*, 138(4), pp. 358–361. doi: https://doi.org/10.1061/(ASCE)HY.1943-7900.0000509

Belzner, F. *et al.* (2017) *Piano key and labyrinth weirs at German waterways: recent and future research of the BAW*. London: CRC Press.

Cicero, G. *et al.* (2013) *Experimental and numerical study of the hydraulic performance of a trapezoidal piano key weir.* London: CRC Press.

Ghodsian, M. and Sohrabzadeh Anzani, H. (2023) ‘Experimental study on flow over rectangular piano key weirs with slopped side crests’, *Modares Civil Engineering Journal*, 23(2), pp. 167–175. doi: http://dx.doi.org/10.22034/23.2.11

Kheir-Abadi, M.A. *et al.* (2020) ‘An experimental study of the effects of the parapet walls geometry on the discharge coefficient of trapezoidal piano key weirs’, *Flow Measurement and Instrumentation*, 73, p. 101742. doi: https://doi.org/10.1016/j.flowmeasinst.2020.101742

Kumar, M. *et al.* (2020) ‘Experimental study and modelling discharge coefficient of trapezoidal and rectangular piano key weirs’, *Applied Water Science*, 10(1), p. 43. doi: https://doi.org/10.1007/s13201-019-1104-8

Leite Ribeiro, M. et al. (2009) ‘Hydraulic capacity improvement of existing spillways – design of a piano key weirs’, *Proceedings of the 23rd Congress of the International Commission on Large Dams (ICOLD)*. Brasília, Brazil, 25–29 May. Paris: ICOLD, pp. 100–118. Available at: https://infoscience.epfl.ch/entities/publication/7cd17737-c1cf-4ee4-b725-0bfc7dc340ef (Accessed date: 25 April 2025).

Machiels, O. (2012) *Experimental study of the hydraulic behaviour of Piano Key Weirs*. PhD thesis. Université de Liège. Available at: https://hdl.handle.net/2268/128006 (Accessed date: 25 April 2025).

Machiels, O. *et al.* (2013) 'Parapet wall effect on piano key weir efficiency', *Journal of Irrigation and Drainage Engineering*, 139(6), pp. 506–511. doi: https://doi.org/10.1061/(ASCE)IR.1943-4774.0000566

Machiels, O. *et al*. (2011) 'Experimental observation of flow characteristics over a piano key weir', *Journal of Hydraulic Research*, 49(3), pp. 359–366. doi: https://doi.org/10.1080/00221686.2011.567761

Rezaei Ahvanooei, A., Mousavi, S.-F. and Karami, H. (2019) 'Improvement of hydraulic performance of nonlinear piano-key weirs in plan', Modares *Civil Engineering Journal*, 19(4), pp. 71–82. Available at: http://mcej.modares.ac.ir/article-16-26776-en.html (Accessed date: 25 April 2025).

Sangsefidi, Y. *et al*. (2021) 'Hydrodynamics and free-flow characteristics of piano key weirs with different plan shapes', *Water*, 13(15), p. 2108. doi: https://doi.org/10.3390/w13152108

Shaker, M. *et al.* (2023) 'Novel techniques to study the effect of parapet wall geometry on the performance of piano key weirs', Water, 15(13), p. 2307. doi: https://doi.org/10.3390/w15132307

Sohrabzadeh, H. and Ghodsian, M. (2022) 'Experimental study of the effect of sidewall slope over the triangular PK weir', *Journal of Hydraulics*, 17(4), pp. 17–30. doi: https://doi.org/10.30482/jhyd.2022.326177.1581

Sohrabzadeh Anzani, H. and Ghodsian, M. (2024b) 'Laboratory study on the discharge coefficient of rectangular piano key weir with partially sloped crest', *Journal of Hydraulics*, 19(1), pp. 1–12. doi: https://doi.org/10.30482/jhyd.2022.354595.1613

Yarmohammadi, B. and Ahadian, J. (2016) 'Experimental study of flow hydraulic in piano key weirs at different parapet wall', *Irrigation Sciences and Engineering*, 39(4), pp. 47–58. doi: https://doi.org/10.22055/jise.2016.12495