

Original paper

Developing a framework for compatibility analysis of predictive climatic variables distribution with reference evapotranspiration in probabilistic analysis of water requirement

Ehsan Fadaei Kermani*, Gholam Abbas Barani, Mohamad Javad Khanjani

Department of Civil Engineering, Shahid Bahonar University, Kerman, Iran.

ARTICLE INFO

Article history:

Received 11 March 2014

Received in revised form 18 May 2014

Accepted 13 June 2014

Keywords:

Compatibility analysis

Reference evapotranspiration

Chow analysis

Probability distribution function

ABSTRACT

In this paper, a new framework has been developed for compatibility analysis of predictive climatic variables distribution with reference evapotranspiration (ET_o) in probabilistic analysis of water requirement. Initially, measured monthly meteorological data of four cities of Iran including Kerman, Shiraz, Ramsar and Babolsar synoptic weather station recorded from 1961 to 2003 were considered based on De Martonne climate classification. Then monthly ET_o was calculated using FAO-Penman-Moentith (FAO-PM), and optimum Probability distribution function (PDF) was determined. The Chow method has been used for frequency analysis, and compatibility analysis was implemented on results. Based on the results, the Generalized Pareto (GP) was selected as optimum PDF for ET_o. Results showed that the optimum PDF for minimum and maximum temperature, solar radiation and relative humidity is GP which had compatibility with ET_oPDF. Eventually, obtained results in compatibility analysis framework were confirmed using Correlation analysis. The proposed methodology developed in this research has application capability in probability scheduling of design water requirement, and can be utilized to optimize probability estimate of water requirement.

© 2014 Razi University-All rights reserved

1. Introduction

Given the problems associated with the development of new water resources due to environmental impact and costs, the effective methods of water resources management need the correction of water use efficiency. The first step in this way is to understand the water requirement based on the time and location (Yoo et al. 2008). Evapotranspiration data are permanently required for irrigation design or planning at the desired probability level. Therefore this study attempted to develop a new framework for analyzing the distribution compatibility of predictive climatic variables of reference evapotranspiration for the probabilistic analysis of water requirement. To evaluate the efficiency of the present method and other methods of plant reference evapotranspiration under different climatic conditions, a basic research has been performed under the supervision of American Society of Civil Engineers (ASCE). In this study, to assess the validity of these methods, the efficiency of twenty different methods has been studied and analyzed in comparison with the lysimeter data from eleven regions with variable climate conditions. The results from analyzing the performance of various methods have proposed the FAO Penman Moentith (FAO-PM) method as the only standard method (Kin et al. 2010); moreover, this method has also been suggested as the reference method by the International Committee on Irrigation and Drainage (ICID) and the World Meteorological Organization (WMO).

In the standard design of agricultural water resources development, locations-plot formulas including California and Weibull have been proposed to find the irrigation requirements and alternate design (Yoo et al. 2008).

Probability distribution functions have been mostly used in the hydrological discussions and less of them have been used in the

debates of ET_o probability determination and water requirements. The two-parameter gamma distribution has been used for daily rainfall and flooding (Oksoy 1999). The review of the studies in this area (Ricciardi et al. 2003; Abida and Ellouze 2008; Wright et al. 2000) shows that only limited research has been done to investigate the distribution governing ET_o; however, no research has been done on the compatibility of predictive variables of water requirements with ET_o, despite its importance. Moreover, there is no framework on the use of probabilistic distributions governing the water requirements and ET_o as a suitable tool in the analysis. Therefore, the main objective of this study is to develop a multi-stage framework for analyzing the compatibility of predictive variables with water requirements in probabilistic estimates and the calibration results of the proposed method.

2. Materials and methods

In this study, at first the values of ET_o are computed using FAO-PM method. Then the best statistical distribution function of ET_o and meteorological variables of monthly time series are specified using Kolmogorov Smirnov method and probability plot correlation coefficient. Finally, the compatibility analysis and assessment of the methodology are performed.

2.1. The study area

In this study, it has been used from data of synoptic stations located in four cities of Iran with different climates based on Domarten climate classification method from 1961 to 2003. Details of the study stations have been listed in Table 1. Table 2 also shows the characteristics of the variables used in this study.

*Corresponding author Email: ehsanhard@gmail.com

Table 1. Characteristics of the studied stations.

Station	Longitude	Latitude	Elevation above sea level (m)	Climate
Kerman	56o 58´ E	30o 15´ N	1749	Arid
Shiraz	52o 36´ E	29o 32´ N	1484	Semi-Arid
Babolsar	52o 39´ E	36o 43´ N	-21	Semi-Humid
Ramsar	50o 40´ E	36o 54´ N	-20	Humid

Table 2. Annual average (43 years) of meteorological parameters at the studied stations.

Station	Min temperature (oC)	Max temperature (oC)	Humidity (%)	Sunshine hours	Wind speed (m/s)
Kerman	3.5	21	32.1	7.4	1.9
Shiraz	6.5	22.4	38.7	8.3	1.7
Babolsar	13.2	20.9	82.8	4.9	0.9
Ramsar	12.6	19.4	84.2	3.6	1.1

2. 2. Period Analysis

Probabilistic estimation of water demand with different return periods requires period analysis. Usually in water resources and hydrological studies, Chow (1951) method is mainly used. In this paper, in order to determine the optimal probability distribution function of ETo and meteorological variables, 15 Typical probability distribution functions including Beta (B), Johnson SB (JSB), Gumbel Min (GMin), Gumbel Max (Gmax), Normal (NOR), Gamma (GAM), Gen Gamma (GG), Log Gamma (LG), Lognormal (LN), Pareto (PAR), Person 5 (P5), Weibull (WBU), Gen Extreme Value (GEV), Gen Pareto (GP) and Log Person 3 (LP3) have been used. It is worth noting that in this study, period analysis was used to estimate the meteorological variables and ETo values.

Period analysis method used is as follows: (1) Creating monthly time series for each station, (2) estimation of parameters in each PDF using the method of moments or square error, (3) ETo estimation using optimized PDF and Chow Periodic factor method, (4) selecting and defining optimized PDF using Goodness-of-Fit tests including Kolmogorov – Smirnov (K-S) tests and Probability Plot Correlation Coefficient (PPCC) (Chow 1951; Vogel and McMartin 1991; Vose 2010; Temesgen et al. 2005).

3. Results and discussion

The results of this study have been presented in this section. At first the tests were carried out on Evapotranspiration variables are provided, and then the results of predictive variables are presented. Finally the results of Harmonic Analysis and validation of method will be presented.

3.1. The results of period analysis of reference Evapotranspiration

In this study, ETo values have been calculated for each region using FAO-PM method. The statistical characteristics of monthly mean values of ETo have been given for all stations on Table 3. According to results, the maximum value of ETo happens in Kerman station which has dry climate, and its minimum value happens in Ramsar station located in a very humid climate. Parameters of 15 probability distribution functions also have been estimated using the moment method, and related K-S values have been calculated and ranked for each distribution. The probability distribution functions related to K-S values have been given on Table 4.

Table 3. Statistical characteristics of monthly ETo.

Station	Minimum	Maximum	Mean	Standard Deviation	Variation Coefficient	Skewness
Kerman	0.7	7.4	3.6	1.7	0.1	0.2
Shiraz	1.1	7.4	3.8	1.7	0.1	0.1
Babolsar	0.6	5.4	2.4	1.4	0.1	0.4
Ramsar	0.8	5.4	2.1	1.2	0.1	0.5

Table 4. Values of KS index in distribution functions.

Station	GP	GG	GEV	LP3	WBU	LN	P5	NOR	GAM
Kerman	0.028*	0.052	0.070	0.081	0.078	0.078	0.075	0.082	0.090
Shiraz	0.048*	0.070	0.088	0.099	0.098	0.099	0.099	0.097	0.108
Babolsar	0.065	0.064	0.101	0.085	0.089	0.089	0.091	0.117	0.106
Ramsar	0.046*	0.070	0.084	0.076	0.087	0.087	0.084	0.117	0.086

*critical values at 5 % significance level

According to Table 4, the Probability Plot Correlation Coefficient (PPCC) values are calculated which have been shown on Table 5. To determine the optimized PDF, mean and standard deviations values have been calculated for each PDF. If the mean value is the highest and standard deviation is the lowest, PDF can accurately create sample data. According to table 5, maximum value of mean and minimum value of standard deviation are related to the GG function, but because this function was not significant based on the K-S test, therefore, the GP function is selected as the optimal function. Figure 1 shows the probabilities for the GP distribution function for all stations which show high correlations and estimations for all stations except Babolsar. In Tables 6 and 7, respectively, the statistical properties of GP distribution for all stations have been given for ETo data and GP parameters. As indicated on Table 6, the GP distribution on ETo data at the Babolsar station has the highest variation coefficient and

skewness coefficient, representing that this distribution cannot create acceptable results in this climate but is the best distribution function.

The GP probability distribution function has been chosen as optimized PDF is calculated as follows (Castillo and Hadi 1997):

$$F(x, k, \sigma) = \begin{cases} 1 - \left(1 - \frac{kx}{\sigma}\right)^{\frac{1}{k}}; & k \neq 0, \sigma > 0 \\ 1 - \exp\left(-\frac{x}{\sigma}\right); & k = 0, \sigma > 0 \end{cases} \quad (1)$$

where, σ and k are scale and shape parameters of the distribution function.

Table 5. Optimized probability distribution functions based on PPCC index.

Station	GP	GG	GEV	LP3	WBU	LN	P5	NOR	GAM
Kerman	0.999	0.982	0.677	0.282	0.963	0.781	0.663	0.978	0.945
Shiraz	0.997	0.974	0.776	0.462	0.957	0.817	0.762	0.969	0.941
Babolsar	0.861	0.973	0.786	0.434	0.975	0.847	0.774	0.959	0.963
Ramsar	0.997	0.978	0.788	0.419	0.984	0.859	0.776	0.963	0.974
Mean	0.964	0.977	0.757	0.399	0.970	0.826	0.744	0.967	0.956
Standard Deviation	0.068	0.004	0.053	0.080	0.012	0.035	0.054	0.008	0.015

Table 6. Statistical characteristics of optimized probability distribution functions based on ETo (mm per month).

Station	Minimum	Maximum	Mean	Variance	Standard Deviation	Variation Coefficient	Skewness
Kerman	1	7	3.6	2.8	1.7	0.6	0.2
Shiraz	1	7	3.8	2.8	1.7	0.4	0.1
Babolsar	0	5.2	1.3	3.6	1.9	1.4	7.1
Ramsar	0.5	5.6	2.1	1.3	1.2	0.6	0.6

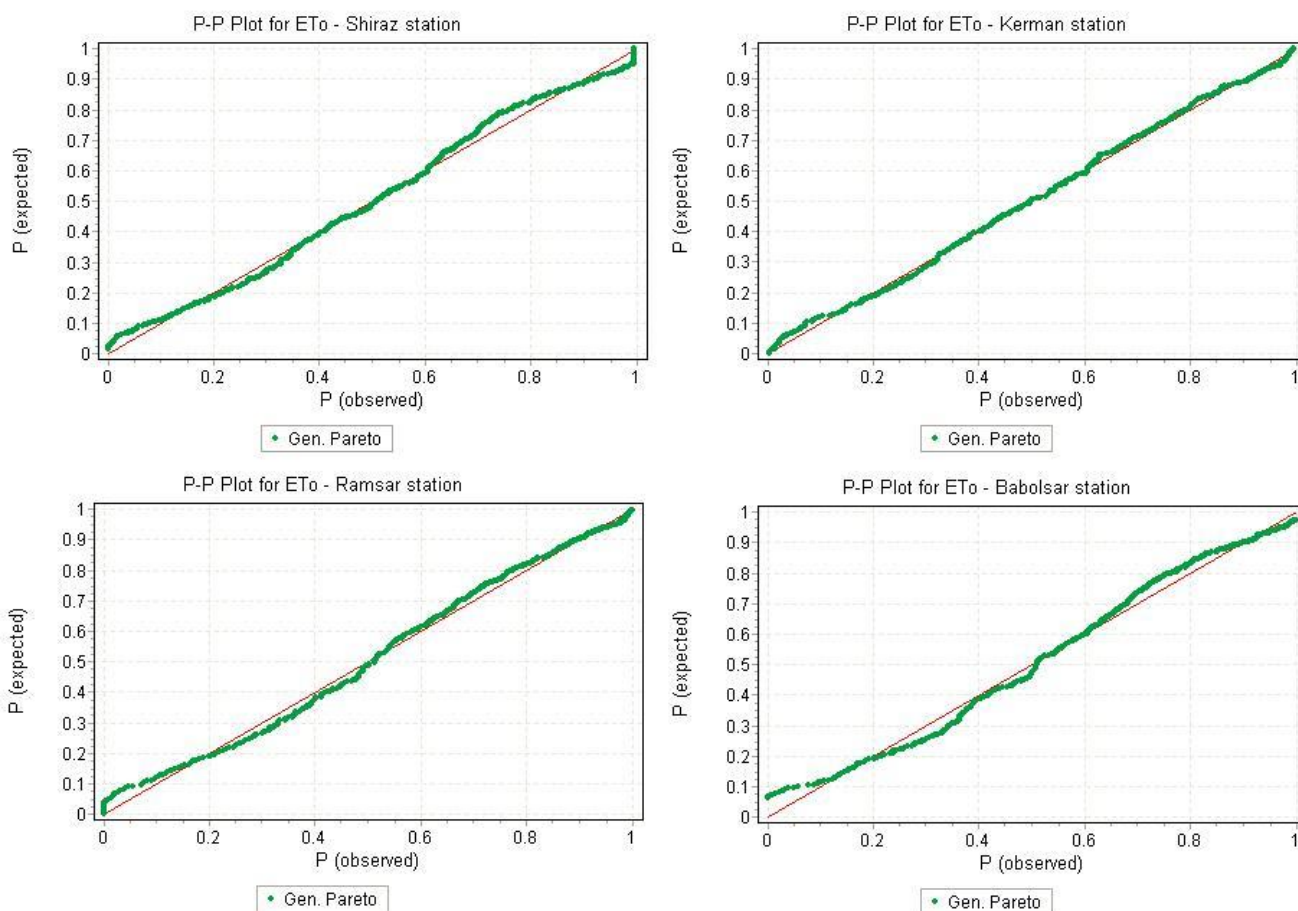


Fig. 1. Probability Plots of GP in study stations.

Table 7. Specifications of parameters for optimal distribution function at each station.

Station	Distribution parameters		
	k	σ	μ
Kerman	-0.8	4.8	1
Shiraz	-0.9	5.4	1
Babolsar	0.3	1	0
Ramsar	-0.5	2.3	0.5

3.2. Period analysis results of predictive variables

In the next step, the probability distribution functions of different variables in minimum temperature, maximum temperature, mean relative humidity, radiation and wind speed were also investigated. In this section, the same mentioned framework used in the ETo analysis was used and the Framework was developed for each of the variables. Table 8 presents the K-S values of significant functions, and the PPCC values for the optimized functions have been given on Table 9.

According to the mean and standard deviation values, it can be found out that the GP function for two variables of maximum temperature and light intensity has better answer than two variables of minimum temperature and relative humidity. The PDF parameters of GP and GEV are given on Tables 10 and 11. GEV Probability distribution function is calculated as follows:

$$t(x) = \begin{cases} \left(1 + k \left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{k}} & \text{if } k \neq 0 \\ e^{-\frac{x-\mu}{\sigma}} & \text{if } k = 0 \end{cases} \quad (2)$$

where μ is the location parameter, σ is the scale parameter, and k is the shape parameter. Figure 2 shows the probability plot graph (PP plot) for variable wind speed and efficiency of the GEV cumulative distribution function for all the stations. It can be seen that the results of the GEV distribution function for the variable of wind speed are reliable which can create better results than other distribution functions for wind speed.

Table 8. K-S values of distribution for the variables.

Station	Minimum temperature (GP)	Maximum temperature (GP)	Mean relative humidity (GP)	Radiation (GP)	wind speed (GEV)
Kerman	0.050	0.039	0.041	0.046	0.039
Shiraz	0.029	0.034	0.050	0.047	0.025
Babolsar	0.054	0.041	0.066	0.048	0.024
Ramsar	0.047	0.034	0.067	0.049	0.055

Table 9. PPCC values of distribution for the variables.

Station	Minimum temperature (GP)	Maximum temperature (GP)	Mean relative humidity (GP)	Radiation (GP)	wind speed (GEV)
Kerman	0.851	0.995	0.998	0.998	0.994
Shiraz	0.998	0.996	0.996	0.997	0.836
Babolsar	0.995	0.997	0.821	0.996	0.855
Ramsar	0.996	0.998	0.789	0.997	0.981
Mean	0.960	0.996	0.901	0.997	0.916
Standard Deviation	0.072	0.001	0.111	0.0008	0.082

Table 10. Parameters of the GP distribution function for the variables.

Station	Minimum temperature			Maximum temperature			Mean relative humidity			Radiation		
	K	σ	μ	k	σ	μ	K	σ	μ	k	σ	μ
Kerman	0.25	1	0	-0.009	23.81	9.08	-0.48	30.22	11.87	-1.12	24.52	6.005
Shiraz	-0.88	15.91	-1.98	-1.004	23.38	7.59	-0.53	36.80	14.73	-1.25	26.85	6.95
Babolsar	-0.96	22.78	1.49	-0.99	24.24	8.64	0.25	1	0	-0.64	13.70	5.05
Ramsar	-0.98	22.73	1.03	-1.004	23.38	7.58	0.25	1	0	-0.52	10.43	4.91

Table 11. Parameters of the GEV distribution function for variable of speed wind.

Station	Distribution parameters		
	k	Σ	μ
Kerman	-0.32	0.95	1.64
Shiraz	0.5	1	0
Babolsar	0.5	1	0
Ramsar	-0.02	0.43	0.87

3. Harmonic analysis results

It was found from the results obtained in the previous sections that optimized PDF values related to ETo for different climates has been the GP function. Moreover, the optimized PDF for four meteorological variables including minimum temperature, maximum temperature, relative humidity and radiation intensity is GP function that shows the

probability distribution of these variables is harmonic with ETo. But the probability distribution of wind speed (GEV) was different from the ETo. Fig. 3 shows the harmonic variables process with ETo for Kerman station located in the dry climate. It can be seen that the variables crated similar trends wit ETo values so that the ETo values increase with increasing of these variables values and decrease with decreasing of them. It is to be mentioned that the variable of wind speed does not have similar trends with ETo as shown on Fig. 4.

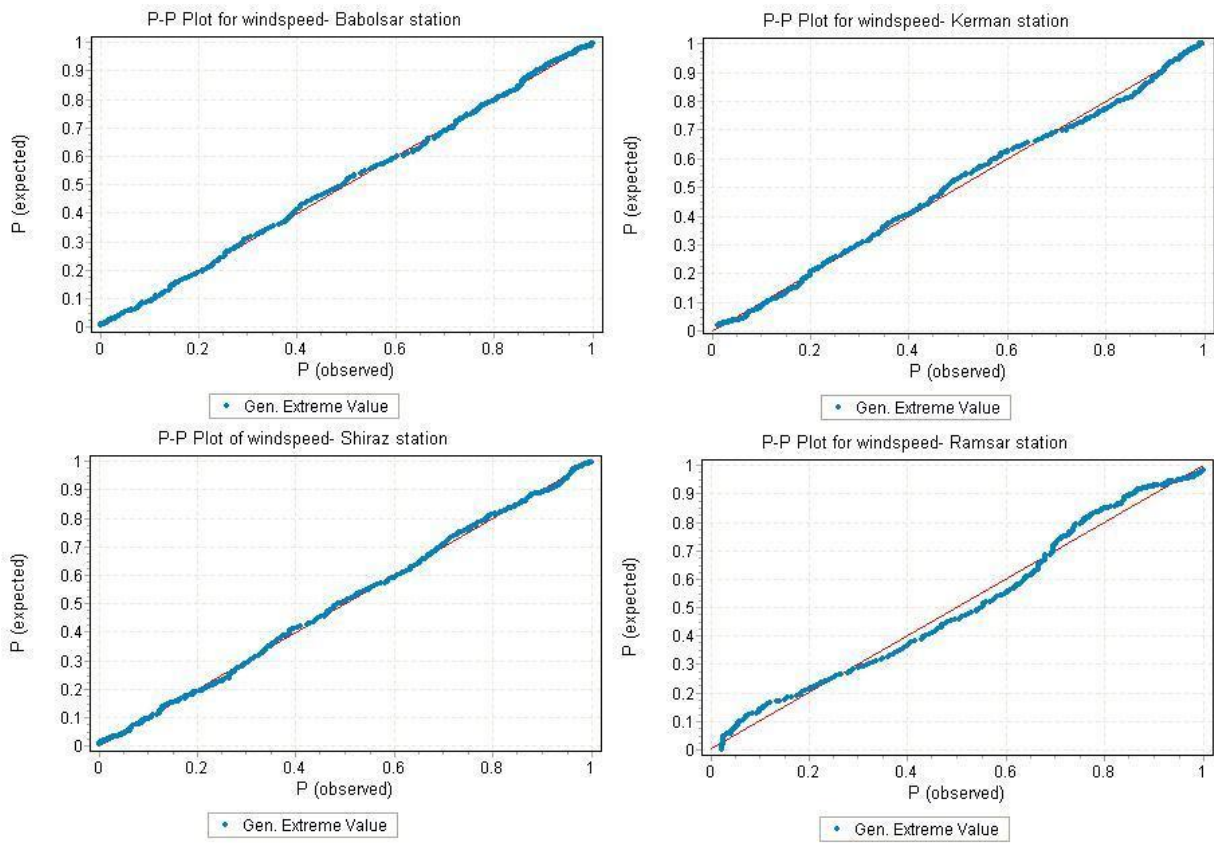


Fig. 2. GEV probability plots of variable wind speed for the study stations.

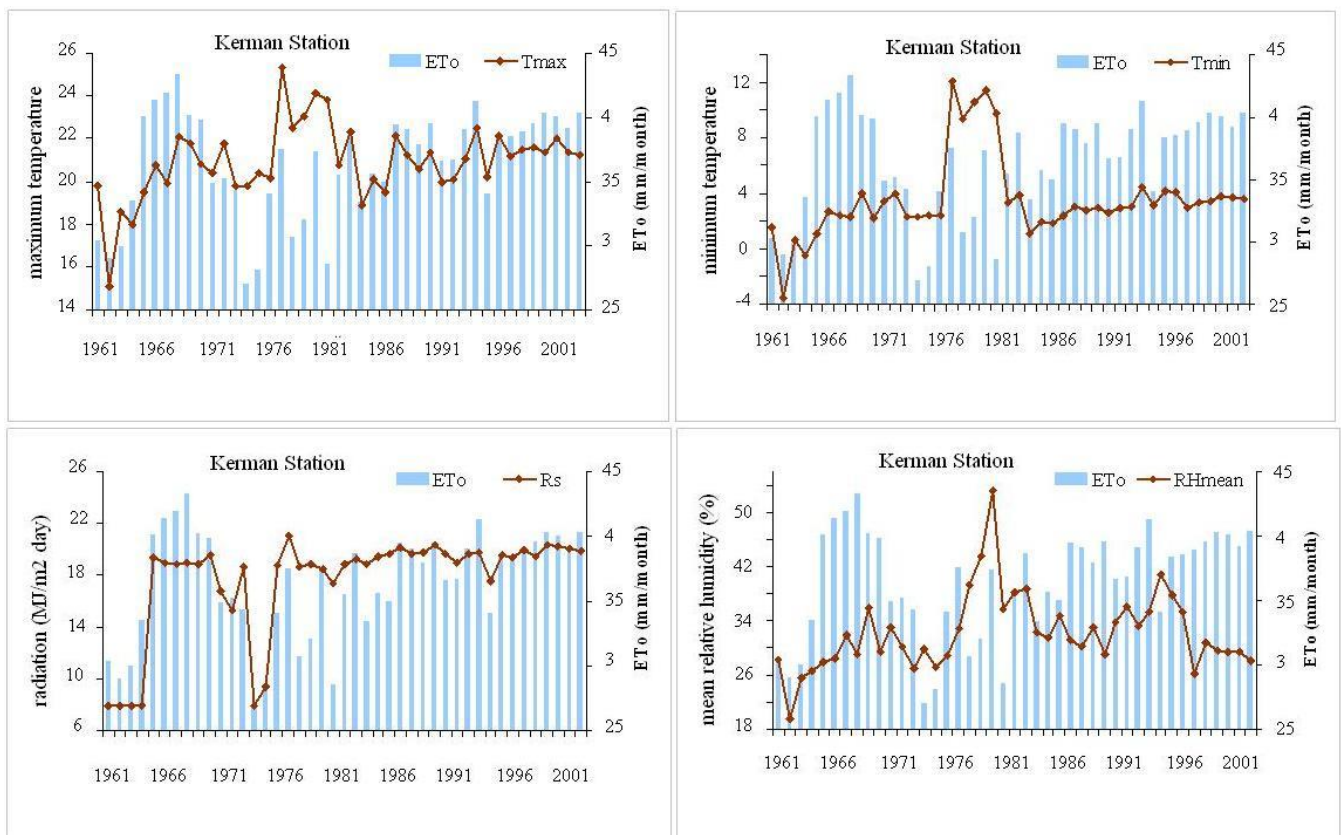


Fig. 3. Process of variables with harmonic distribution of ET0.

3.4. Results validation

Using the present results in practical aspects require confirmation and verification with other independent methods. In this section the results of the harmonic analysis of predictive variables for water requirement have been validated. Accordingly, it is expected to obtain a better correlation between the parameters of the harmonic probability distribution with different distribution parameters compared to ETo distribution. Therefore, the verification of the results was done by calculating the coefficients of determination (R2) between measured parameters at stations and calculated ETo values. The

relationship of reference Evapotranspiration with the variables is presented in Figs. 5 to 8 for Kerman station. It can be seen that the three variables including minimum temperature, maximum temperature and radiation intensity have reasonable coefficient of determination associated with ETo values. The R2 value of the variable wind speed and ETo is very low (R2= 0.22) which represented no compatibility between the two variables in terms of the probability distribution. It can be find out that the results of the analysis variables are compatible with ETo to approve and confirm the framework presented in this research.

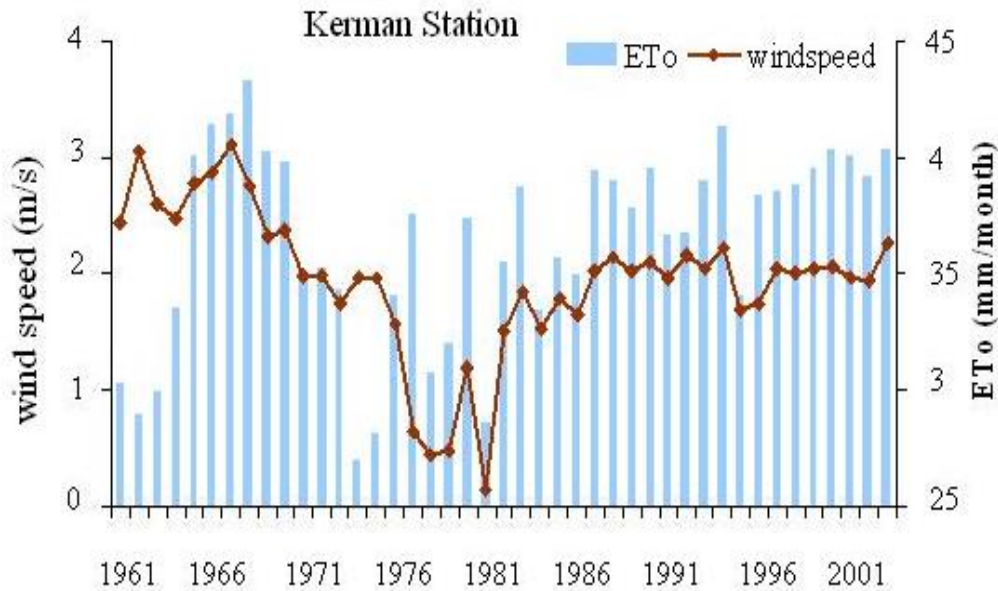


Fig. 4. Process of wind speed variable with non-harmonic distribution of ETo.

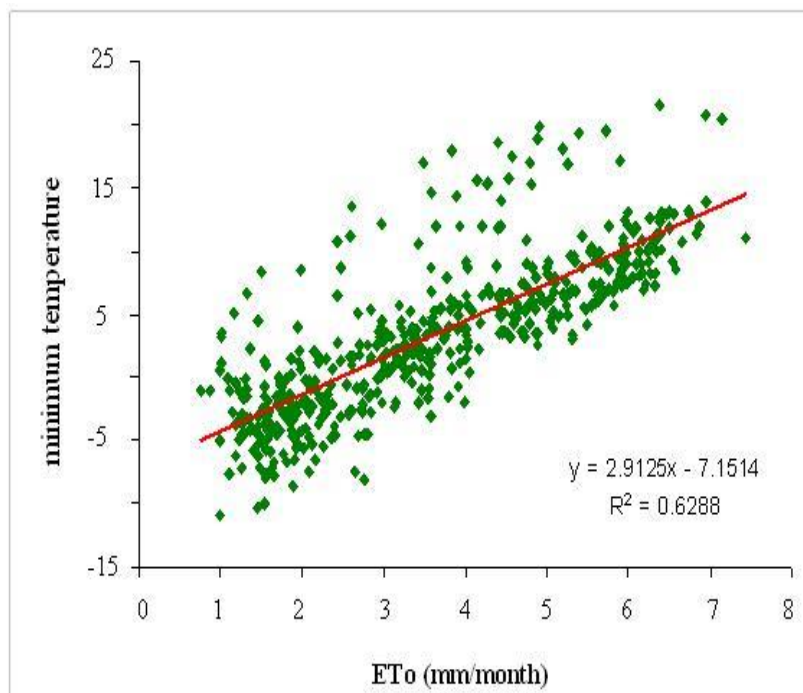


Fig. 5. Relationship of reference evapotranspiration with minimum temperature variable.

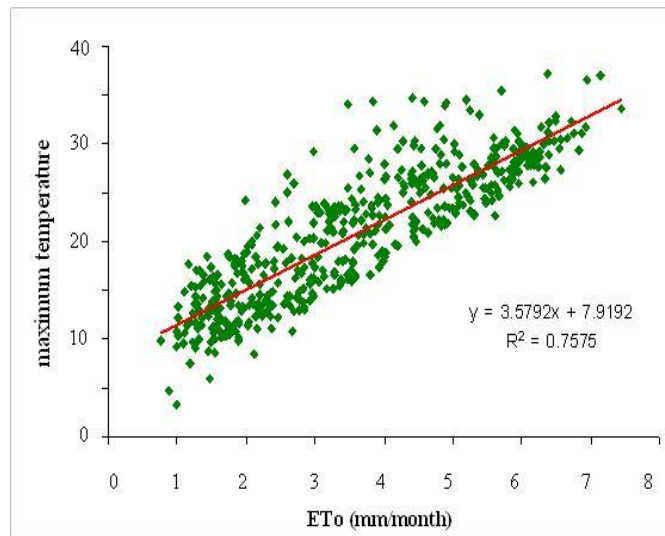


Fig. 6. Relationship of reference evapotranspiration with maximum temperature variable.

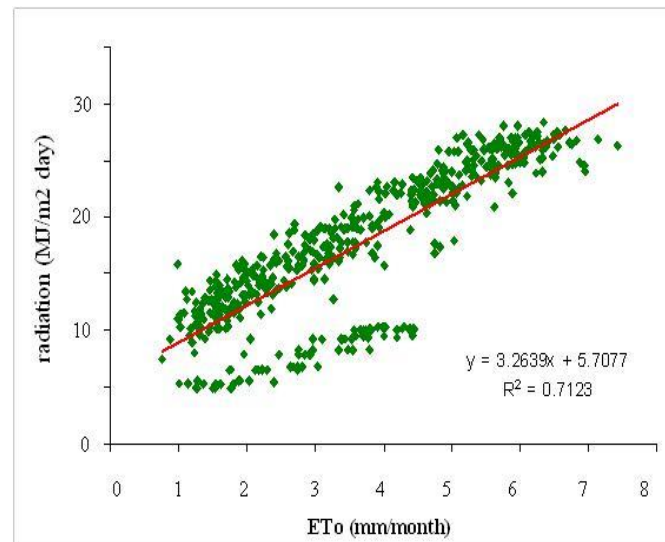


Fig. 7. Relationship of reference evapotranspiration with radiation variable.

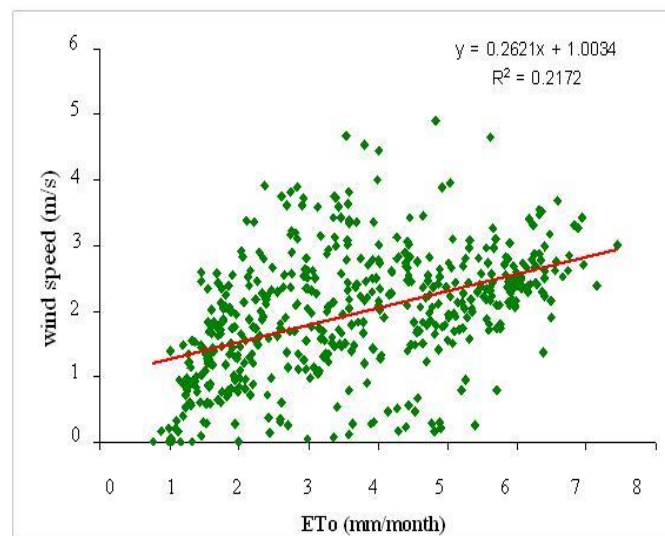


Fig. 8. Relationship of reference evapotranspiration with wind speed variable

4. Conclusions

Given the importance of probabilistic estimates of reference evapotranspiration (ET_o) in irrigation management, in this paper, it has been attempted to develop a framework for finding the best probability distribution function (PDF) in determining the average monthly ET_o and the most important meteorological variables affecting ET_o including minimum and maximum temperature, relative humidity, wind speed, sunshine hours and radiation in four different climates of Iran. Results showed that the best PDF for monthly ET_o

values is the GP function. Moreover, it was found that the best PDF for variables of minimum and maximum temperature, relative humidity, radiation intensity is also the GP function which was compatible with the distribution function of ET_o. Finally the validity of the presented framework was evaluated using correlation analysis which showed the developed framework can be utilized as a suitable tool in the probabilistic analysis of water requirement. The results of this study can be used as a guide for calculation of ET_o at different climates, and the approach developed in this study can be applied in probabilistic water demand planning projects.

References

- Abida H., Ellouze M., Probability distribution of flood flows in Tunisia, *Hydrology and Earth System Sciences* 12 (2008) 703-714.
- Castillo E., Hadi A.S., Fitting the Generalized Pareto distribution to data, *American Statistical Association* 92 (1997) 1609-1619.
- Chow V.T., A general formula for hydrologic frequency analysis. *Transactions, American Geophysical Union* 32, 1951.
- Kim S., Shin H., Kim T., Heo J.H., Derivation of the probability plot correlation coefficient test statistics for the generalized logistic distribution, *International Workshop Advances in Statistical Hydrology*, Taormina, Italy, May (2010) 23-25.
- ksoy H., Use of Gamma distribution in hydrological analysis, *Turkish Journal of Engineering and Environmental Sciences* 24 (1999) 419-428.
- icciardi K.L., Pinder G.F., Belitz K., Comparison of the lognormal and beta distribution functions to describe the uncertainty in permeability, *Hydrology* 313 (2005) 248-256.
- Temesgen B., Eching S., Davidoff B., Frame K., Comparison of some reference evapotranspiration equations for California, *Journal of Irrigation and Drainage Engineering* 131 (2005) 73-84.
- Vogel R.M., McMartin D.E., Probability plot goodness-of-fit and skewness estimation procedures for the Pearson type distribution. *Water Resources Research* 27 (1991) 3149-3158.
- Vose D. Fitting distributions to data and why you are probably doing it wrong, 2010, Available in www.voftware.com.
- Wright J.L., Allen, R.G., Howell, T.A. Conversion between evapotranspiration references and methods. *Proceedings of the 4th National Irrigation Symposium*, ASAE, Phoenix, AZ, 2000.
- Yoo S.H., Choi, J.Y., Jang, M.W., Estimation of design water requirement using FAO Penman-Monteith and optimal probability distribution function in South Korea, *Agricultural Water Management* 95 (2008) 845-853.